CHAPTER 10

CIRCLES

(A) Main Concepts and Results

Circle, radius, diameter, chord, segment, cyclic quadrilateral.

- Equal chords of a circle (or of congruent circles) subtend equal angles at the centre,
- If the angles subtended by the chords of a circle (or of congruent circles) at the centre (or centres) are equal, then the chords are equal,
- The perpendicular drawn from the centre of the circle to a chord bisects the chord,
- The line drawn through the centre of a circle bisecting a chord is perpendicular to the chord,
- There is one and only one circle passing through three given non-collinear points,
- Equal chords of a circle (or of congruent circles) are equidistant from the centre (or centres),
- Chords equidistant from the centre of a circle are equal in length,
- If two chords of a circle are equal, then their corresponding arcs are congruent and conversely, if two arcs are congruent, then their corresponding chords are equal,
- Congruent arcs of a circle subtend equal angles at the centre,
- The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle,
- Angles in the same segment of a circle are equal,
- If a line segment joining two points subtends equal angles at two other points lying on the same side of the line containing the line segment, then the four points are concyclic,

- The sum of either pair of opposite angles of a cyclic quadrilateral is 180º,
- If the sum of a pair of opposite angles of a quadrilateral is 180º, the quadrilateral is cyclic.

(B) Multiple Choice Questions

Write the correct answer :

Sample Question 1: In Fig. 10.1, two congruent circles have centres O and O′. Arc AXB subtends an angle of 75º at the centre O and arc A′ Y B′ subtends an angle of 25º at the centre O′. Then the ratio of arcs A X B and A′ Y B′ is:

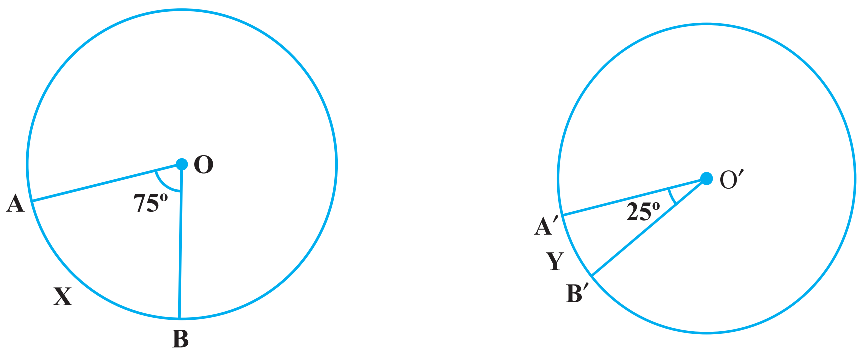

Fig. 10.1

(A) 2 : 1 (B) 1 : 2 (C) 3 : 1 (D) 1 : 3

Solution : Answer (C)

Sample Question 2 : In Fig. 10.2, AB and CD are two equal chords of a circle with centre O. OP and OQ are perpendiculars on chords AB and CD, respectively. If ∠POQ = 150º, then ∠APQ is equal to

(A) 30º (B) 75º

(C) 15º (D) 60º

Solution : Answer (B)

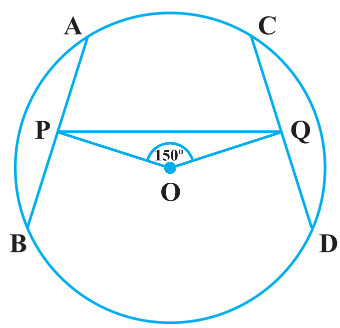

Fig. 10.2

EXERCISE 10.1

1. AD is a diameter of a circle and AB is a chord. If AD = 34 cm, AB = 30 cm, the distance of AB from the centre of the circle is :

(A) 17 cm (B) 15 cm (C) 4 cm (D) 8 cm

2. In Fig. 10.3, if OA = 5 cm, AB = 8 cm and OD is perpendicular to AB, then CD is equal to:

(A) 2 cm (B) 3 cm

(C) 4 cm (D) 5 cm

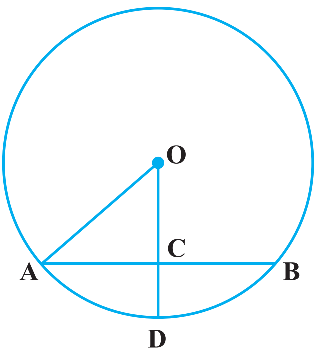

Fig. 10.3

3. If AB = 12 cm, BC = 16 cm and AB is perpendicular to BC, then the radius of the circle passing through the points A, B and C is :

(A) 6 cm (B) 8 cm

(C) 10 cm (D) 12 cm

4. In Fig.10.4, if ∠ABC = 20º, then ∠AOC is equal to:

(A) 20º (B) 40º (C) 60º (D) 10º

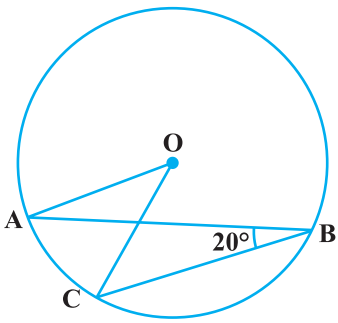

Fig. 10.4

5. In Fig.10.5, if AOB is a diameter of the circle and AC = BC, then ∠CAB is equal to:

(A) 30º (B) 60º

(C) 90º (D) 45º

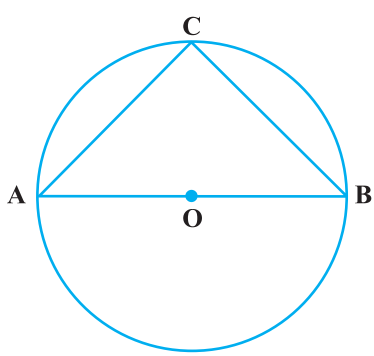

Fig. 10.5

6. In Fig. 10.6, if $\angle OAB = 40^\circ$, then $\angle ACB$ is equal to :

(A) 50° (B) 40° (C) 60° (D) 70°

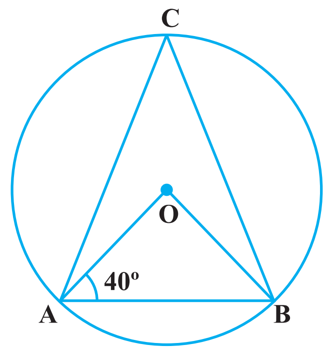

Fig. 10.6

7. In Fig. 10.7, if $\angle DAB = 60^\circ$, $\angle ABD = 50^\circ$, then $\angle ACB$ is equal to:

(A) 60° (B) 50° (C) 70° (D) 80°

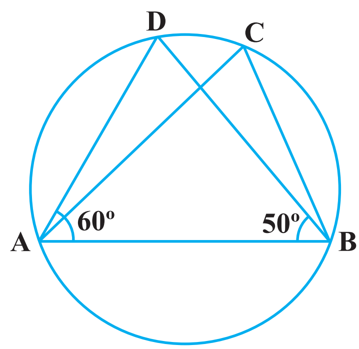

Fig. 10.7

8. ABCD is a cyclic quadrilateral such that AB is a diameter of the circle circumscribing it and $\angle ADC = 140^\circ$, then $\angle BAC$ is equal to:

(A) 80° (B) 50°

(C) 40° (D) 30°

9. In Fig. 10.8, BC is a diameter of the circle and $\angle BAO = 60^\circ$. Then $\angle ADC$ is equal to :

(A) 30° (B) 45°

(C) 60° (D) 120°

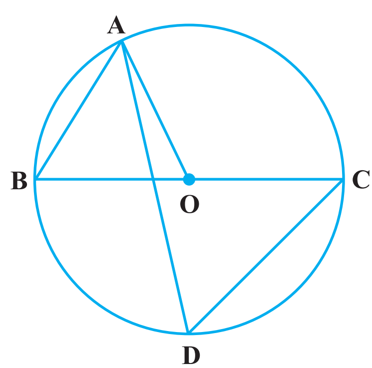

Fig. 10.8

10. In Fig. 10.9, $\angle AOB = 90^\circ$ and $\angle ABC = 30^\circ$, then $\angle CAO$ is equal to:

(A) 30° (B) 45° (C) 90° (D) 60°

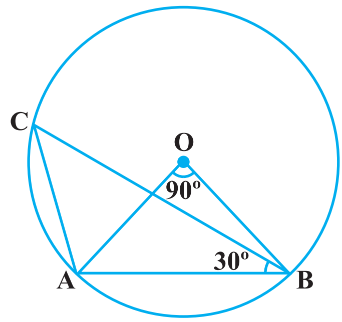

Fig. 10.9

(C) Short Answer Questions with Reasoning

Write **True** or **False** and justify your answer.

Sample Question 1: The angles subtended by a chord at any two points of a circle are equal.

Solution : False. If two points lie in the same segment (major or minor) only, then the angles will be equal otherwise they are not equal.

Sample Questions 2 : Two chords of a circle of lengths 10 cm and 8 cm are at the distances 8.0 cm and 3.5 cm, respectively from the centre.

Solution: False. As the larger chord is at smaller distance from the centre.

EXERCISE 10.2

Write **True** or **False** and justify your answer in each of the following:

1. Two chords AB and CD of a circle are each at distances 4 cm from the centre. Then AB = CD.

2. Two chords AB and AC of a circle with centre O are on the opposite sides of OA. Then $\angle OAB = \angle OAC$.

3. Two congruent circles with centres O and O′ intersect at two points A and B. Then $\angle AOB = \angle AO'B$.

4. Through three collinear points a circle can be drawn.

5. A circle of radius 3 cm can be drawn through two points A, B such that AB = 6 cm.

6. If AOB is a diameter of a circle and C is a point on the circle, then $AC^2 + BC^2 = AB^2$.

7. ABCD is a cyclic quadrilateral such that $\angle A = 90°$, $\angle B = 70°$, $\angle C = 95°$ and $\angle D = 105°$.

8. If A, B, C, D are four points such that $\angle BAC = 30°$ and $\angle BDC = 60°$, then D is the centre of the circle through A, B and C.

9. If A, B, C and D are four points such that $\angle BAC = 45°$ and $\angle BDC = 45°$, then A, B, C, D are concyclic.

10. In Fig. 10.10, if AOB is a diameter and $\angle ADC = 120°$, then $\angle CAB = 30°$.

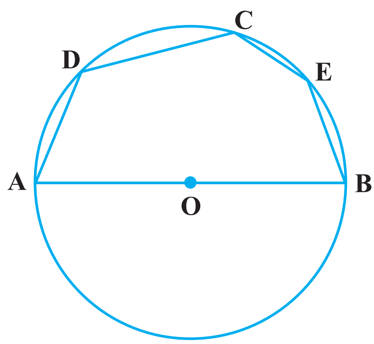

Fig. 10.10

(D) Short Answer Questions

Sample Question 1 : In Fig. 10.11, AOC is a diameter of the circle and arc AXB = $\frac{1}{2}$ arc BYC. Find $\angle BOC$.

Solution :

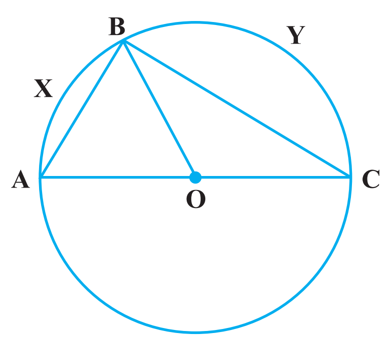

Fig. 10.11

As $\quad$ arc AXB = $\frac{1}{2}$ arc BYC,

$$\angle AOB = \frac{1}{2} \angle BOC$$

Also $\quad \angle AOB + \angle BOC = 180°$

Therefore, $\frac{1}{2} \angle BOC + \angle BOC = 180°$

or $\angle BOC = \frac{2}{3} \times 180^\circ = 120^\circ$

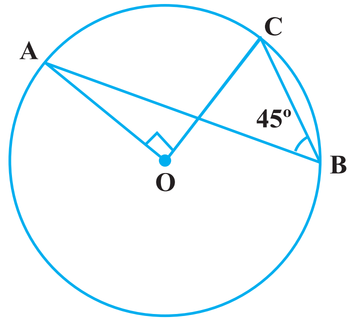

Fig. 10.12

Sample Question 2 : In Fig. 10.12, $\angle ABC = 45^\circ$, prove that $OA \perp OC$.

Solution : $\angle ABC = \frac{1}{2} \angle AOC$

i.e., $\angle AOC = 2\angle ABC = 2 \times 45^\circ = 90^\circ$

or $OA \perp OC$

EXERCISE 10.3

1. If arcs AXB and CYD of a circle are congruent, find the ratio of AB and CD.

2. If the perpendicular bisector of a chord AB of a circle PXAQBY intersects the circle at P and Q, prove that arc PXA $\cong$ Arc PYB.

3. A, B and C are three points on a circle. Prove that the perpendicular bisectors of AB, BC and CA are concurrent.

4. AB and AC are two equal chords of a circle. Prove that the bisector of the angle BAC passes through the centre of the circle.

5. If a line segment joining mid-points of two chords of a circle passes through the centre of the circle, prove that the two chords are parallel.

6. ABCD is such a quadrilateral that A is the centre of the circle passing through B, C and D. Prove that

$$\angle CBD + \angle CDB = \frac{1}{2} \angle BAD$$

7. O is the circumcentre of the triangle ABC and D is the mid-point of the base BC. Prove that $\angle BOD = \angle A$.

8. On a common hypotenuse AB, two right triangles ACB and ADB are situated on opposite sides. Prove that $\angle BAC = \angle BDC$.

9. Two chords AB and AC of a circle subtends angles equal to 90° and 150°, respectively at the centre. Find $\angle BAC$, if AB and AC lie on the opposite sides of the centre.

10. If BM and CN are the perpendiculars drawn on the sides AC and AB of the triangle ABC, prove that the points B, C, M and N are concyclic.

11. If a line is drawn parallel to the base of an isosceles triangle to intersect its equal sides, prove that the quadrilateral so formed is cyclic.

12. If a pair of opposite sides of a cyclic quadrilateral are equal, prove that its diagonals are also equal.

13. The circumcentre of the triangle ABC is O. Prove that $\angle OBC + \angle BAC = 90^\circ$.

14. A chord of a circle is equal to its radius. Find the angle subtended by this chord at a point in major segment.

15. In Fig.10.13, $\angle ADC = 130^\circ$ and chord BC = chord BE. Find $\angle CBE$.

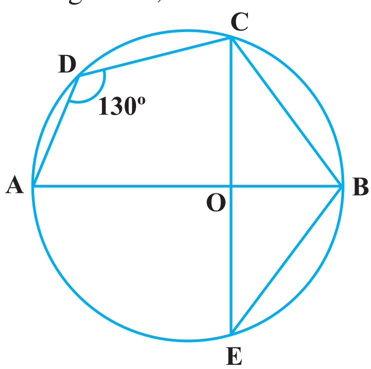

Fig. 10.13

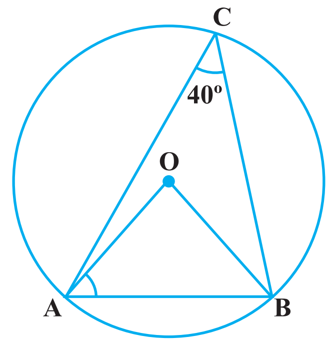

Fig. 10.14

16. In Fig.10.14, $\angle ACB = 40^\circ$. Find $\angle OAB$.

17. A quadrilateral ABCD is inscribed in a circle such that AB is a diameter and $\angle ADC = 130^\circ$. Find $\angle BAC$.

18. Two circles with centres O and O′ intersect at two points A and B. A line PQ is drawn parallel to OO′ through A(or B) intersecting the circles at P and Q. Prove that PQ = 2 OO′.

19. In Fig.10.15, AOB is a diameter of the circle and C, D, E are any three points on the semi-circle. Find the value of $\angle ACD + \angle BED$.

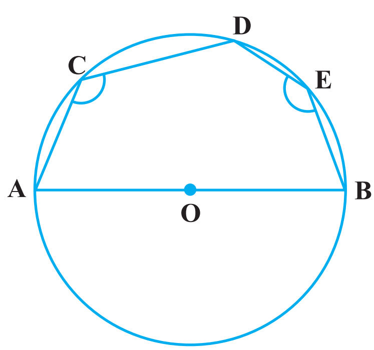

Fig. 10.15

20. In Fig. 10.16, ∠OAB = 30º and ∠OCB = 57º. Find ∠BOC and ∠AOC.

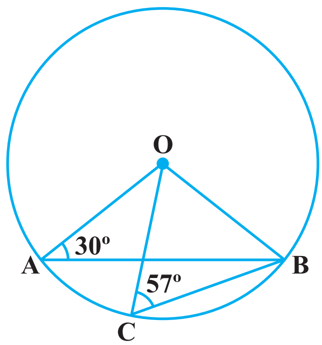

Fig. 10.16

(E) Long Answer Questions

Sample Question 1 : Prove that two circles cannot intersect at more than two points.

Solution : Let there be two circles which intersect at three points say at A, B and C. Clearly, A, B and C are not collinear. We know that through three non-collinear points A, B and C one and only one circle can pass. Therefore, there cannot be two circles passing through A, B and C. In other words, the two circles cannot intersect at more than two points.

Sample Question 2 : Prove that among all the chords of a circle passing through a given point inside the circle that one is smallest which is perpendicular to the diameter passing through the point.

Solution : Let P be the given point inside a circle with centre O. Draw the chord AB which is perpendicular to the diameter XY through P. Let CD be any other chord through P. Draw ON perpendicular to CD from O. Then ΔONP is a right triangle (Fig.10.17). Therefore, its hypotenuse OP is larger than ON. We know that the chord nearer to the centre is larger than the chord which is farther to the centre. Therefore, CD > AB. In other words, AB is the smallest of all chords passing through P.

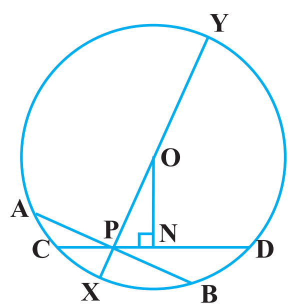

Fig. 10.17

EXERCISE 10.4

1. If two equal chords of a circle intersect, prove that the parts of one chord are separately equal to the parts of the other chord.

2. If non-parallel sides of a trapezium are equal, prove that it is cyclic.

3. If P, Q and R are the mid-points of the sides BC, CA and AB of a triangle and AD is the perpendicular from A on BC, prove that P, Q, R and D are concyclic.

4. ABCD is a parallelogram. A circle through A, B is so drawn that it intersects AD at P and BC at Q. Prove that P, Q, C and D are concyclic.

5. Prove that angle bisector of any angle of a triangle and perpendicular bisector of the opposite side if intersect, they will intersect on the circumcircle of the triangle.

6. If two chords AB and CD of a circle AYDZBWCX intersect at right angles (see Fig.10.18), prove that arc CXA + arc DZB = arc AYD + arc BWC = semi-circle.

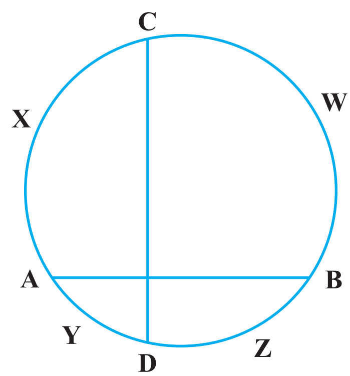

Fig. 10.18

7. If ABC is an equilateral triangle inscribed in a circle and P be any point on the minor arc BC which does not coincide with B or C, prove that PA is angle bisector of $\angle$BPC.

8. In Fig. 10.19, AB and CD are two chords of a circle intersecting each other at point E. Prove that

$\angle AEC = \frac{1}{2}$ (Angle subtended by arc CXA at centre + angle subtended by arc DYB at the centre).

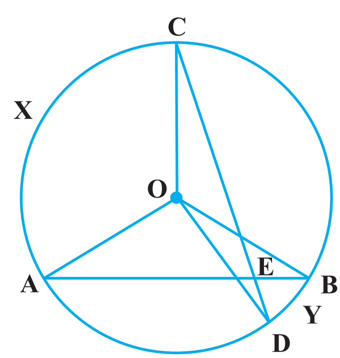

Fig. 10.19

9. If bisectors of opposite angles of a cyclic quadrilateral ABCD intersect the circle, circumscribing it at the points P and Q, prove that PQ is a diameter of the circle.

10. A circle has radius $\sqrt{2}$ cm. It is divided into two segments by a chord of length 2 cm. Prove that the angle subtended by the chord at a point in major segment is 45º.

11. Two equal chords AB and CD of a circle when produced intersect at a point P. Prove that PB = PD.

12. AB and AC are two chords of a circle of radius r such that AB = 2AC. If p and q are the distances of AB and AC from the centre, prove that $4q^2 = p^2 + 3r^2$.

13. In Fig. 10.20,O is the centre of the circle, $\angle$BCO = 30°. Find x and y.

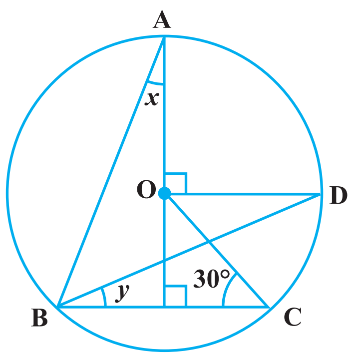

Fig. 10.20

14. In Fig. 10.21, O is the centre of the circle, BD = OD and CD $\perp$ AB. Find $\angle$CAB.

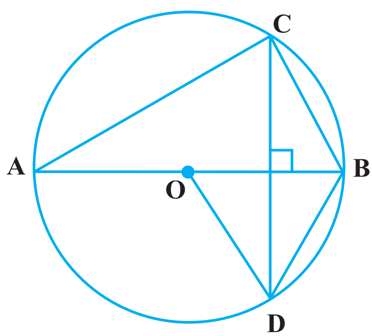

Fig. 10.21