Biyani's Think Tank

Concept based notes

Engineering Mechanics

B.Tech. First Semester

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Preface

I am glad to present this book, especially designed to serve the needs of the students. The book has been written keeping in mind the general weakness in understanding the fundamental concept of the topic. The book is self-explanatory and adopts the "Teach Yourself" style. It is based on question-answer pattern. The language of book is quite easy and understandable based on scientific approach.

Any further improvement in the contents of the book by making corrections, omission and inclusion is keen to be achieved based on suggestions from the reader for which the author shall be obliged.

I acknowledge special thanks to Mr. Rajeev Biyani, *Chiarman*,Dr. Sanjay Biyani, *Director (Acad.)* Biyani Group of Colleges & Mr. Manish Biyani, Director Biyani International Institute of Engineering & Technology, who is the backbone and main concept provider and also have been constant source of motivation throughout this endeavour. We also extend our thanks to M/s. Hastlipi, Omprakash Agarwal/Sunil Kumar Jain, Jaipur, who played an active role in co-ordinating the various stages of this endeavour and spearheaded the publishing work.

I look forward to receiving valuable suggestions from professors of various educational institutions, other faculty members and the students for improvement of the quality of the book. The reader may feel free to send in their comments and suggestions to the under mentioned address.

Author

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Chapter.No.	Name of Topic
1	System of forces, Fundamental laws of mechanics, Composition of forces, Free body diagram, Lami's theorem, Rectilinear motion, plane curvilinear motion, Projectile motion, Newton's law of motion, D'Alembert's principle.
2	Moments and couple, Varignon's theorem, condition of equilibrium. Impulse, momentum, Impulse – Momentum relation, Impact.
3	Types of support and loading, reaction, Analysis of simple trusses by methods of joints and method of sections.
4	Laws of Coulomb friction, Ladder, Wedges, Belt friction and rolling, Principle of virtual work and its applications.
5	Location of centroid and center of gravity, area moment of inertia, mass moment of inertia.
6	Work, energy (Potential, Kinetic and Spring), Work – Energy relation, Law of conservation of energy, Constrained motion of connected particles

Syllabus
Engineering Mechanics

Unit - I
• System of forces, Fundamental laws of mechanics, Composition of forces
• Free body diagram, Lami's theorem
• Moments and couple, Varignon's theorem, condition of equilibrium
• Types of support and loading, reaction, Analysis of simple trusses by methods of joints and method of sections

Unit - II
. Laws of Coulomb friction, Ladder, Wedges, Belt friction and rolling
. Principle of virtual work and its applications

Unit - III
• Location of centroid and center of gravity, area moment of inertia, mass moment of inertia
• Law of machines, Variation of mechanical advantages, efficiency, reversibility of machine
• Pulleys, wheel and axle, wheel and differential axle
• Transmission of power through belt and rope

Unit - IV
Kinematics of Particle
. Rectilinear motion, plane curvilinear motion, Projectile motion
. Constrained motion of connected particles
Dynamics of Particle and Rigid Body
• Newton's law of motion
• D'Alembert's principle

Unit - V
Work and Energy
. Work, energy (Potential, Kinetic and Spring)
. Work – Energy relation
. Law of conservation of energy
Impulse and Momentum
• Impulse, momentum
• Impulse – Momentum relation, Impact

Chapter - 1

Introduction :-

In this course on Engineering Mechanics, we shall be learning about mechanical interaction between bodies. That is we will learn how different bodies apply forces on one another and how they then balance to keep each other in equilibrium. That will be done in the first part of the course. So in the first part we will be dealing with STATICS. In the second part we then go to the motion of particles and see how does the motion of particles get affected when a force is applied on them. We will first deal with single particles and will then move on to describe the motion of rigid bodies.

Q.1 What is Force?

Ans. :- Force is the action of one body on another. It may be defined as an action which changes the state of rest or of uniform motion of body. For representing the force acting on the body, the magnitude of the force, its point of action and direction of its action should be known.

According to Newton`s second law of motion, we can write force as

$$F = ma = mass*(length/time^2)$$

There are different type of forces such as gravitational, frictional, magnetic, inertia or those caused by mass and acceleration.

Q.2 What is Force system?

Ans. :- A force system is the collection of forces acting on a body in one or more planes. According to the relative position of the line of action of the forces, the forces may be classified as : Collinear, Coplanar, Concurrent, Coplanar concurrent, Non-coplanar concurrent, Coplanar non-concurrent, Non-coplanar non-concurrent.

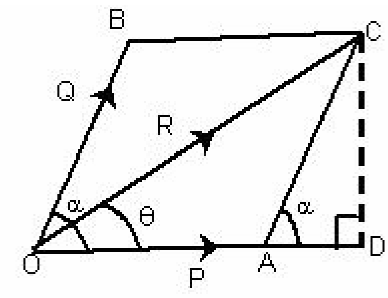
A force system has resultant forces, which is a single force which can be replaced a system of forces and produce the same effect on the body.

Q.3 What is Parallelogram law of Forces?

Ans. :- If two forces, acting at a point, are represented in magnitude and direction by the two sides of a parallelogram drawn from one of its angular points, their resultant is represented both in magnitude and direction by the diagonal of the parallelogram passing through that angular point.

Resultant Force: If two or more forces P, Q, S, act upon a rigid body and if a single force, R, can be found whose effect upon the body is the same as that of the forces P, Q, S, … this single force R is called the resultant of the other forces. Resultant of forces acting in the same direction (same straight line) is equal to their sum.

Magnitude and Direction of the Resultant of Two Forces: Let OA and OB represent the forces P and Q acting at a point O and inclined to each other at an angle a then the resultant R and direction 'q' (shown in figure) will be given by R = √P^2+Q^2+2PQcosα



and tan θ = Qsinα/P+Qcosα

Case (i): If P = Q, then tanq = tan (α/2) => θ = α/2

Case (ii): If the forces act at right angles, so that
α = 90°, we have R = √P^2+Q^2 and tanθ = Q/P

Q.4 Two forces of 100kN and 50kN are acting simultaneously at a point, Find the magnitude and direction of the resultant if the angle between them is 60 degree?

Sol. :- Here F1 = 100kN, F2 = 50kN, θ = 60

Using parallelogram law of forces

R = √(F1^2+F2^2+2F1.F2cosθ)

R = √(100^2 + 50^2 + 2*100*50*cos60)

R = 1323kN

The direction of R is

tan α = F2 sinθ/(F1 + F2 cosθ)

tan α = 50 sin60/(100 + 50 cos60)

α = tan^{-1}(√3/5) = 19.1 degree

Q.5 The resultant of two forces P and Q is R. If Q is doubled, R is doubled and when Q is reversed, R is again doubled, show that P : Q :R :: √2: √3:√2.

Sol: Let a be the angle between the forces P and Q. Now from the given conditions, we have

$$R^2 + P^2 + Q^2 + 2PQ \cos \alpha \qquad \ldots(1)$$

and $(2R)^2 + P^2 + (2Q)^2 + 2P(2Q)\cos \alpha$

$4R^2 = P^2 + 4Q^2 + 4PQ \cos\alpha$ …(2)

and $(2R)^2 = P^2 + (-Q)^2 + 2P(-Q)\cos\alpha$

=> $4R^2 = P^2+Q^2 - 2PQ \cos\alpha$ …(3)

Adding (1) and (3), we get $2P^2 + 2Q^2 - 5R^2 = 0$ …(4)

Eliminating a from (2) and (3), we get

$P^2 + 2Q^2 - 4R^2 = 0$ …(5)

From (4) and (5), we have

$P^2/-8+10 = Q^2/-5+8 = R^2/4-2$ => $P^2/2 = Q^2/3 = R^2/2$ => P:Q:R:: √2: √3: √2.

Q.6 Explain triangular law of Forces?

Ans. :- Triangle Law of Forces

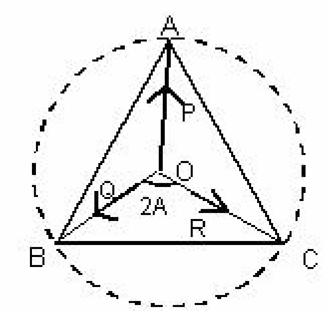
If three forces, acting at a point, are represented in magnitude and direction by the sides of a triangle, taken in order, they will be in equilibrium.

Q.7 Explain Lami`s Theorem?

Ans. :- Lami's Theorem:

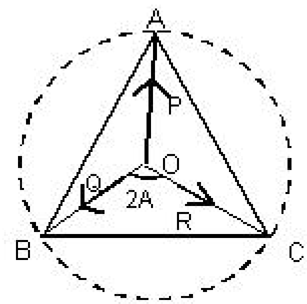
If three forces acting on a particle keep it in equilibrium, each is proportional to the sine of the angle between the other two.

Thus if the forces are P, Q and R and α, β, γ be the angles between Q, R; R, P; and P, Q respectively then if the forces are in equilibrium, we have P/ *aß?*



Q.8 Forces P, Q and R acting along OA, OB and OC, where O is the circumcentre of the triangle ABC are in equilibrium. Show that

$$P/a^2(b^2+c^2-a^2) = Q/b^2(c^2+a^2-b^2) = R/c^2(a^2+b^2-c^2).$$



Sol. :- ∠ BOC = 2A, ∠COA = 2B, ∠AOB = 2C

Applying Lami's Theorem , we get

$$P/\sin 2A = Q/\sin 2B = R/\sin 2C$$

$$\Rightarrow P/2\sin A\cos A = Q/2\sin B\cos B = R/2\sin C\cos C$$

Apply sine rule and cosine rule to get

$$P2bc/a(b^2+c^2-a^2) = Q.2ac/b(c^2+a^2-b^2) = R.2ab/c(a^2+b^2-c^2)$$

$$\Rightarrow P/a^2(b^2+c^2-a^2) = Q./b^2(c^2+a^2-b^2) = R/c^2(a^2+b^2-c^2).$$

Q.9 How resolution of Forces can be done?

Ans. :- RESOLUTION OF FORCES

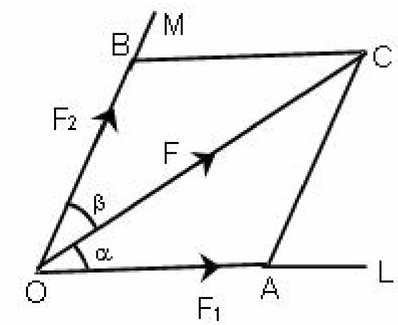
A force may be resolved into two components in an infinite number of ways. The most important case of the resolution of forces occurs when we resolve a force into two components at right angles to one another.

Components of a Force in Two Directions:

Let F be the given force represented in magnitude and direction by OC and let the directions of the two components be along OL and OM.

Also ∠COL = α and ∠COM = β. Then

$F_1 = F\sin\beta/\sin(\alpha+\beta)$ and $F_2 = F\sin\alpha/\sin(\alpha+\beta)$



Q.10 The resultant of two forces P and Q is equal to √3Q and makes an angle of 30° with the direction of P; show that P is either equal to Q or double of Q.

Ans. :- Let $R = Q\sqrt{3}$ be the resultant of two forces and α = angle between the forces P and Q.

Clearly $R\cos 30° = P+Q\cos\alpha$ and $R\sin 30° = Q\sin\alpha$

=> $Q\cos\alpha = R\cos 30° - P$ and $R\sin 30° = Q\sin\alpha$

=> $(R\cos 30° - P)^2 + (R\sin 30°)^2 = Q^2$

=> $R^2 + P^2 - 2PR\cos 30° = Q^2$

=> $3Q^2+P^2 - 2P.Q.\sqrt{3}.\sqrt{3}/2 = Q^2$

=> $P^2-3PQ+2Q^2 = 0$

=> $(P-Q)(P-2Q) = 0$ => $P = Q$ or $2Q$.

Q.11 What is Free Body Diagram?

Ans. :- A free body diagram is a pictorial representation often used by physicists and engineers to analyze the forces acting on a body of interest. A free body diagram shows all forces of all types acting on this body. Drawing such a diagram can aid in solving for the unknown forces or the equations of motion of the body. Creating a free body diagram can make it easier to understand the forces, and torques or moments, in relation to one another and suggest the proper concepts to apply in order to find the solution to a problem. The diagrams are also used as a conceptual device to help identify the internal forces—for example, shear forces and bending moments in beams—which are developed within structures.

Q.12 Explain Newton`s laws of motion?

Ans. :- First law: A body does not change its state of motion unless acted upon by a force. This law is based on observations but in addition it also defines an inertial frame . By definition an inertial frame is that in which a body does not change its state of motion unless acted upon by a force.

Second law: The second law is also part definition and part observation. It gives the force in terms of a quantity called the mass and the acceleration of a particle. It says that a force of magnitude F applied on a particle gives it an acceleration a proportional to the force. In other words

$$F = ma ,$$

where m is identified as the inertial mass of the body.

Third Law: Newton's third law states that if a body A applies a force F on body B , then B also applies an equal and opposite force on A . (Forces do not cancel such other as they are acting on two different objects)

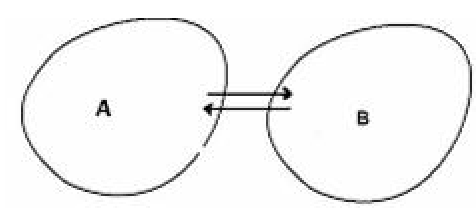
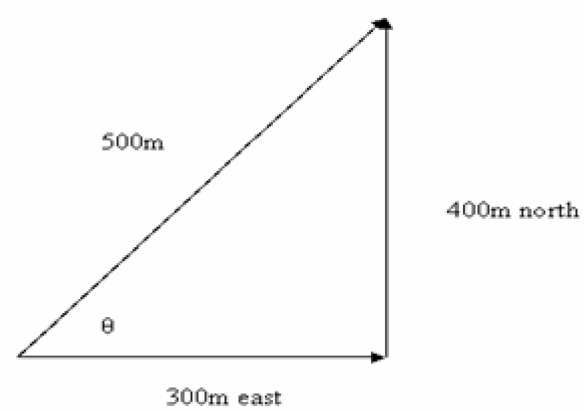


Figure 1

Thus if they start from the position of rest A and B will tend to move in opposite directions. You may ask: if A and B are experiencing equal and opposite force, why do they not cancel each other? This is because - as stated above - the forces are acting on two different objects.

Q.13 A person walks 300m to the east and 400m to the north to reach his friend's house. What is the total displacement of the person, and what is the total distance traveled by him?

Sol. :- Recall that distance is a scalar quantity. Thus the total distance covered is 700m. Displacement, on the other hand, is a vector quantity so to find the net displacement, we add the two vectors to get a displacement of 500m at an angle $\theta = \tan^{-1}\left(\frac{4}{3}\right)$ from east to north (Figure).



Adding displacements of 300m east and 400m north

Figure

Q.14 Two persons are pushing a box so that the net force on the box is 12N to the east If one of the person is applying a force 5N to the north, what is the force applied by the other person.

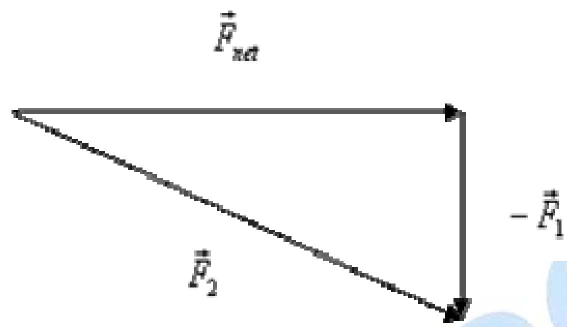
Sol. :- Let the force by person applying 5N be denoted by $\vec{F}_1$ and that by the other person by $\vec{F}_2$. We then have

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$

so that

$$\vec{F}_2 = \vec{F}_{net} - \vec{F}_1$$

Solution for $\vec{F}_2$ is given graphically in figure. The force comes out to be 13N at an angle of $\theta = \tan^{-1}\left(\frac{5}{12}\right)$ from east to south.



Finding force applied by a person when the net force and that applied by one of the persons is given.

Figure

Chapter - 2

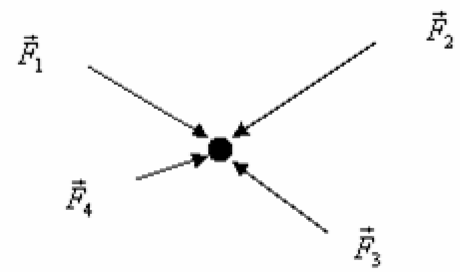
Q.2 What do you understand by Equilibrium?

Ans. :- In the static part when we say that a body is in equilibrium, what we mean is that the body is not moving at all even though there may be forces acting on it. (In general equilibrium means that there is no acceleration i.e., the body is moving with constant velocity but in this special case we take this constant to be zero).

Let us start by observing what all can a force do to a body? One obvious thing it does is to accelerate a body. So if we take a point particle P and apply a force on it, it will accelerate. Thus if we want its acceleration to be zero, the sum of all forces applied on it must vanish. This is the condition for equilibrium of a point particle. So for a point particle the equilibrium condition is

$$\sum_i \vec{F}_i = 0$$

where $\vec{F}_i; i = 1,2,3....$ are the forces applied on the point particle (see figure)

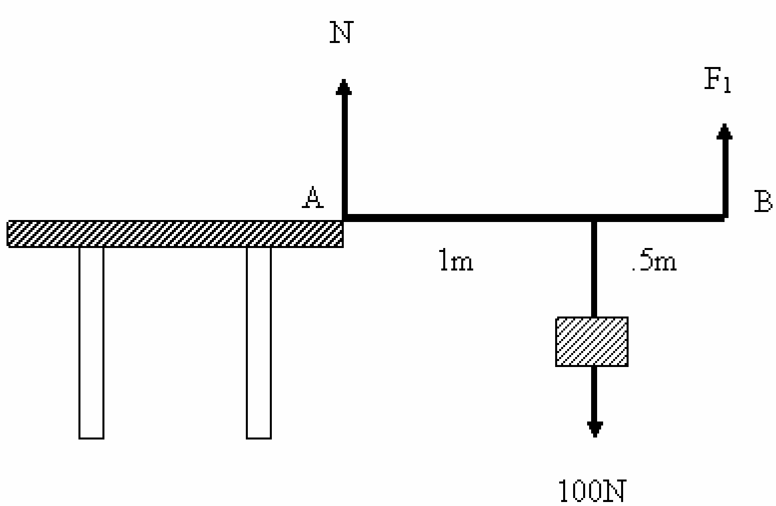


A particle in equilibrium under four forces

Figure

Q.3 A person is holding a 100N weight (that is roughly a 10kg mass) by a light weight (negligible mass) rod AB. The rod is 1.5m long and weight is hanging at a distance of 1m from the end A, which is on a table (see figure). How much force should the person apply to hold the weight?

Ans. :- Let the normal reaction of the table on the rod be N and the force by the point be F_1. Then the two equilibrium conditions give



Figure

$$\sum \vec{F} = 0 \Rightarrow (F_1 + N - 100)\hat{j} = 0 \Rightarrow F_1 + N = 100 \qquad (1)$$

$$\sum \vec{\tau}_{iA} = 0 \Rightarrow \hat{i} \times -100\hat{j} + 1.5\hat{i} \times F_1\hat{j} = 0 \qquad (2)$$

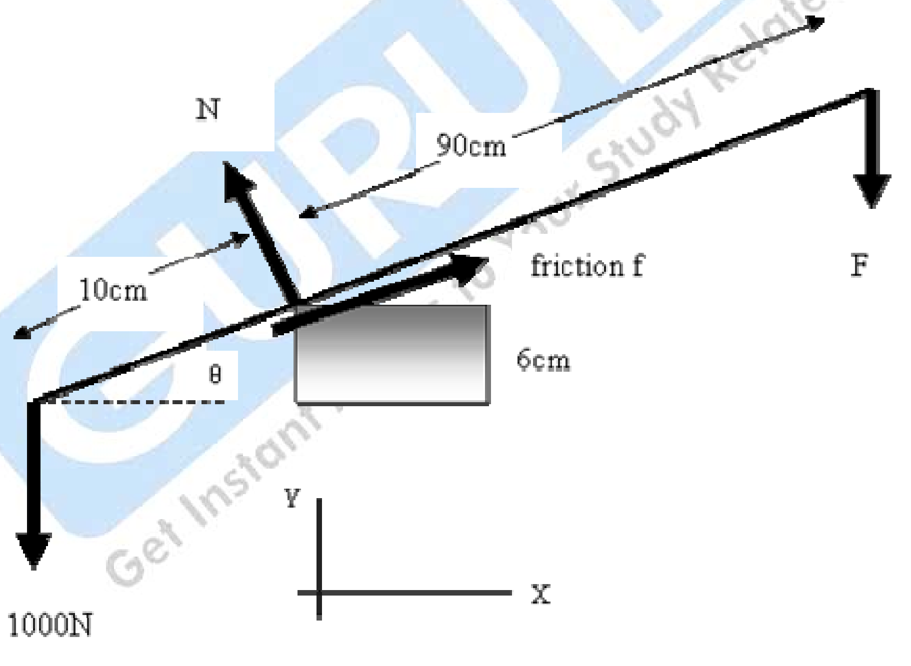
$$-100\hat{k} + F_1 \times 1.5\hat{k} = 0$$

$$\text{or} \quad 1.5F_1 = 100 \quad \Rightarrow \quad F_1 = \frac{100}{1.5} = \left(\frac{200}{3}\right)\text{N}$$

$$\text{and} \quad N = 100 - F_1 = 100 - \frac{200}{3} = \frac{100}{3}\text{N}$$

Q.4 We are trying to lift a 1000N (~100kg mass) weight by putting a light weight but strong rod as shown in the figure using the edge of a brick as the fulcrum. The height of the brick is 6cm. The question we ask is: what is the value of the force applied in the vertical direction that is needed to lift the weight? Assume the brick corner to be rough so that it provides frictional force.

Ans. :-



(Note: If the brick did not provide friction, the force applied cannot be only in the vertical direction as that would not be sufficient to cancel the horizontal component of N). Let us see what happens if the brick offered no friction and we applied a force in the vertical direction. The fulcrum applies a force *N* perpendicular to the rod so if we apply only a vertical force, the rod will tend to slip to the left because of the component of N in that direction. Try it out on a smooth corner and see that it does happen. However, if the friction is there then the rod will not slip. Let us apply the equilibrium conditions in such a situation. The balance of forces gives

$$\sum \vec{F} = 0 \Rightarrow F(N\sin\theta - f\cos\theta)\hat{i} + (N\cos\theta - f\sin\theta - 1000 - F)\hat{j} = 0$$
$$or \quad N\sin\theta = f\cos\theta$$
$$N\cos\theta + f\sin\theta - F - 1000 = 0$$

Let us choose the fulcrum as the point about which we balance the torque. It gives

Then

$$\sum \vec{\tau} = 0 \Rightarrow 0.9\hat{r} \times -F\hat{j} + (-0.1)\hat{r} \times -1000\hat{j} = 0$$
$$\Rightarrow (-\cdot 9\cos\theta F + 100\cos\theta)\hat{k} = 0$$
$$\text{or } F = 111\cdot 11\,\text{N}$$

The normal force and the frictional force can now be calculated with the other two equations obtained above by the force balance equation.

Q.5 What is Torque due to the Force?

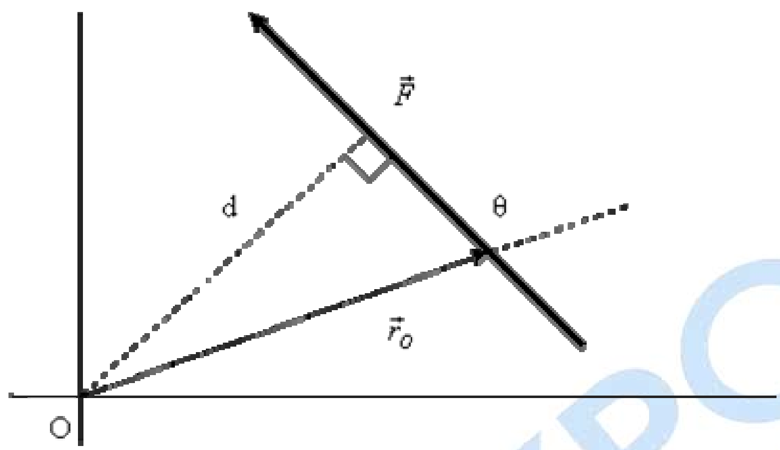
Ans. :- Torque due to a force: Torque about a point due to a force $\vec{F}$ is obtained as the vector product

$$\vec{\tau}_O = \vec{r}_O \times \vec{F}$$
$$= (yF_z - F_y z)\hat{i} + (zF_x - xF_z)\hat{j} + (xF_y - yF_x)\hat{k}$$

where $\vec{r}_O$ is a vector from the point *O* to the point where the force is being applied. Actually $\vec{r}_O$ could be a vector from *O* to any point along the line of action of the force as we will see below. The magnitude of the torque is given as

$$|\vec{\tau}_O| = |\vec{F}||\vec{r}_O|\sin\theta$$

Thus the magnitude of torque is equal to the product of the magnitude of the force and the perpendicular distance $d = |\vec{r}_O|\sin(180° - \theta) = |\vec{r}_O|\sin\theta$ from *O* to the line of action of the force as shown in figure 7 in the plane containing point *O* and the force vector. Since this distance is fixed, the torque due to a force can be calculated by taking vector $\vec{r}_0$ to be any vector from *O* to the line of action of the force. The unit of a torque is Newton-meter or simply Nm.



Torque is equal to the product of the magnitude of the force and its perpendicular distance d from O

Figure 7

Q.6 Let there be a force of 20 N applied along the vector going from point (1,2) to point (5,3). Now find torque due to this force?

Ans. :- So the force can be written as its magnitude times the unit vector from (1,2) to (5,3). Thus

$$\vec{F} = \frac{20\left(4\hat{i} + \hat{j}\right)}{\sqrt{17}}$$

Torque can be calculated about *O* by taking $\vec{r}$ to be either $\left(\hat{i} + 2\hat{j}\right) or \left(5\hat{i} + 3\hat{j}\right)$. As argued above, the answer should be the same irrespective of which $\vec{r}$ we choose. Let us see that. By taking $\vec{r}$ to be $\left(\hat{i} + 2\hat{j}\right)$ we get

$$\vec{\tau}_0 = \frac{\left(\hat{i} + 2\hat{j}\right) \times 20\left(4\hat{i} + \hat{j}\right)}{\sqrt{17}}$$

$$= \frac{20}{\sqrt{17}}(\hat{k} - 8\hat{k}) = -\frac{140\hat{k}}{\sqrt{17}}$$

On the other hand, with $\vec{r} = (5\hat{i} + 3\hat{j})$ we get

$$\vec{\tau}_O = \frac{(5\hat{i} + 3\hat{j}) \times 20(4\hat{i} + \hat{j})}{\sqrt{17}}$$
$$= \frac{20}{\sqrt{17}}(5\hat{k} - 12\hat{k}) = -\frac{140\hat{k}}{\sqrt{17}}$$

Which is the same as that obtained with $\vec{r} = (\hat{i} + 2\hat{j})$. Thus we see that the torque is the same no matter where along the line of action is the force applied. This is known as the **transmissibility** of the force.

Q.7 What is Varignon's theorem?

Ans. :- we know that

$$\vec{\tau}_O = \vec{r} \times \vec{F}$$

where $\vec{r}$ is any vector from the origin to the line of action of the force.

If there are many forces applied on a body then the total moment about *O* is the vector sum of all other moments i.e.

$$\vec{\tau}_O = \sum \vec{r}_{iO} \times \vec{F}_i$$

As a special case if the forces are all applied at the same point *j* then

$$\vec{\tau}_O = \sum \vec{r}_{iO} \times \vec{F}_i = \vec{r}_{jO} \times \sum_i \vec{F}_i$$
$$= \vec{r}_{jO} \times \vec{F}_{net}$$

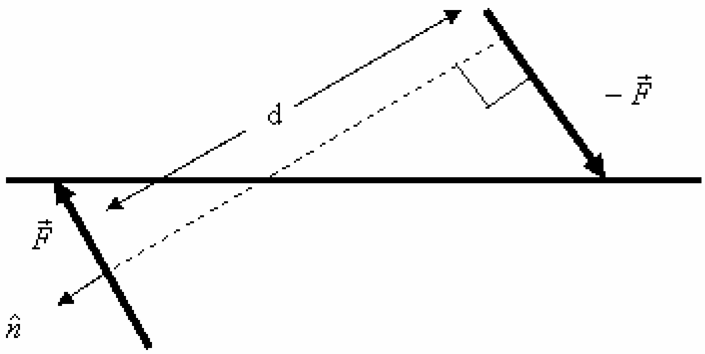
This is known as **Varignon's theorem**. Its usefulness arises from the fact that the torque due to a given force can be calculated as the sum of torques due to its components.

Q.8 What is a Force Couple?

Ans. :- A particular example of the net force being zero is two equal magnitude forces in directions opposite to each other and applied at a distance from one another, as in figure. This is known as a couple and the corresponding torque with respect to any point is given as

$$\vec{\tau}_{couple} = (\hat{n} \times \vec{F})d.$$

where $\hat{n}$ is a unit vector perpendicular to the forces coming out of the space between them and *d* is the perpendicular distance between the forces.



A couple

Figure

Since the net force due to a couple is zero, the only action a couple has on a body is to tend to rotate it. Further the moment of a couple is independent of the origin, and so it can be applied anywhere on the body and it will have the same effect on the body. We can even change the magnitude of the force and alter the distance between them keeping the magnitude of the couple the same. Then also the effect of couple will be the same. Such vectors whose effect remains unchanged irrespective of where they are applied are known as free vectors. Free vectors have a nice property that they can be added irrespective of where they are applied without changing the effect they produce. Thus a couple is a free vector (Is force a free vector?). It is represented by the symbols

Representing a couple

Figure

with the arrows clearly giving the sense of rotation. Keep in mind though that the direction of the couple (in the vector sense) is perpendicular to the plane in which the forces forming the couple are.

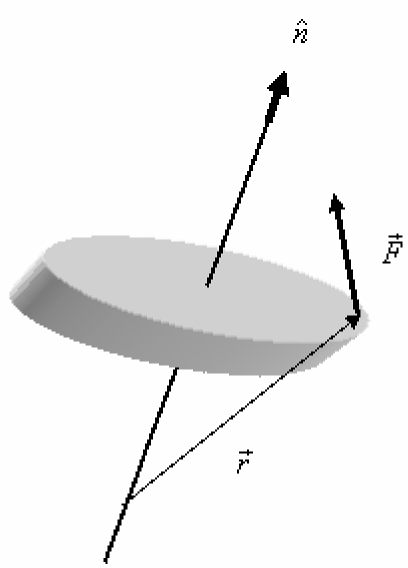
Q.9 Explain Moment of a Force about an Axis?

Ans. :- Moment of force about an axis:

So far we have talked about moment of a force about a point only. However, many a times a body rotates about an axis. This is the situations you have bean studying in you 12th grade. For example a disc rotating about an axis fixed in two fixed ball bearings. In this case what affects the rotation is the component of the torque along the axis, where the torque is taken about a point *O* (the point can be chosen arbitrarily) on the axis as given in figure 11. Thus

$$\vec{\tau}_{aboutaxis} = \hat{n} \cdot (\vec{r} \times \vec{F})$$

where $\hat{n}$ is the unit vector along the axis direction and $\vec{r}$ is the vector from point *O* on the axis to the force $\vec{F}$.



Disc experiencing a torque about an axis

Figure

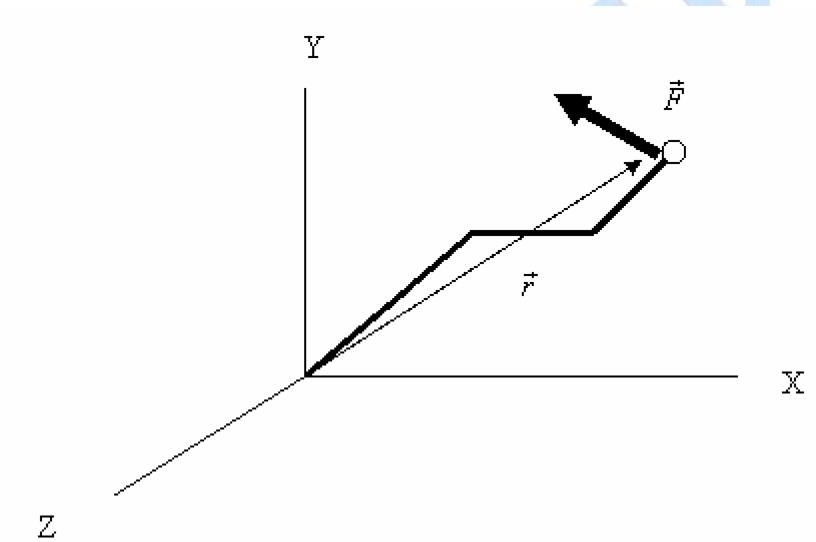
Using vector identities (exercise at the end of Lecture 1), it can also be written as

$\vec{\tau}_{about axis} = (\hat{n} \times \vec{r}) \cdot \vec{F}$

Thus the moment of a force about an axis is the magnitude of the component of the force in the plane perpendicular to the axis times its perpendicular distance from the axis. Thus if a force is pointing towards the axis, the torque generated by this force about the axis would be zero.

Q.10 You must have seen the gear shift handle in old buses. It is of Zigzag shape. Let it be of the shape shown in figure : 60cm at an angle of 45° from the x-axis, 30cm parallel to x-axis and then 30cm again at 45° from the x-axis, all in the x-y plane shown in figure. To change gear a driver applies a force of $\vec{F} = (-5\hat{i} + 5\hat{j} - 2\hat{k})\text{N}$ **on the head of the handle. We want to know what is the equivalent force and moment at the bottom i.e., at the origin of the handle.**

Ans. :-



Figure

For this again we can apply a zero force i.e., ($\vec{F}$ and $-\vec{F}$) at the bottom so that original force and $-\vec{F}$ give a couple moment

$$\tau = \vec{r} \times \vec{F}$$

$$= \left(\frac{60}{\sqrt{2}}\hat{i} + \frac{60}{\sqrt{2}}\hat{j} + 30\hat{i} + \frac{30}{\sqrt{2}}\hat{i} + \frac{30}{\sqrt{2}}\hat{j}\right) \times (-5\hat{i} + 5\hat{j} - 2\hat{k})$$

$$= \left(\frac{90 + 30\sqrt{2}}{\sqrt{2}}\hat{i} + \frac{90}{\sqrt{2}}\hat{j}\right) \times (-5\hat{i} + 5\hat{j} - 2\hat{k})$$

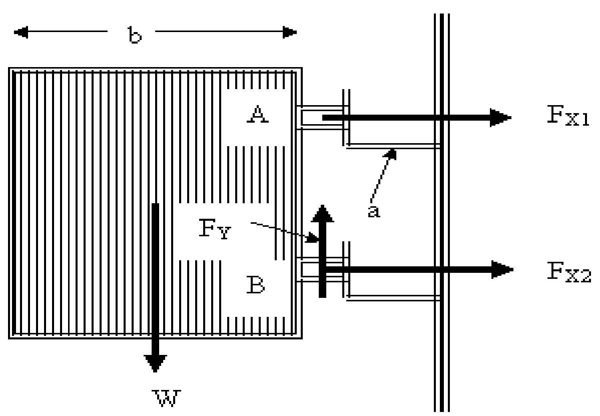
$$= \frac{450 + 150\sqrt{2}}{\sqrt{2}}\hat{k} + \frac{180 + 60\sqrt{2}}{\sqrt{2}}\hat{j} + \frac{450}{\sqrt{2}}\hat{k} - 90\sqrt{2}\hat{i}$$

$$= -90\sqrt{2}\hat{i} + (90\sqrt{2} + 60)\hat{j} + 600\sqrt{2}\hat{k}$$

Thus equivalent force system is a force $\vec{F} = (-5\hat{i} + 5\hat{j} - 2\hat{k})\text{N}$ at the bottom and a couple equal to $-90\sqrt{2}\hat{i} + (90\sqrt{2} + 60)\hat{j} + 600\sqrt{2}\hat{k}$ Nm.

Q.11 You must have seen gates being supported on two supports (see figure). Suppose the weight of the gate is *W* and its width *b*. The supports are protruding out of the wall by *a* and the distance between them is *h*. If the weight of the gate is supported fully by the lower support, find the horizontal forces, vertical forces and the moment load on both the supports.

Ans. :-



A gate supported on two fixed supports

Figure

To solve this problem, let us first find out what are the forces required to keep the gate in balance. The forces applied by the supports on the gate are shown in figure

. Since the weight of the gate is fully supported on the lower support all the vertical force is going to be provided by the lower support only. Thus

$$\sum F_y = 0 \Rightarrow F_Y - W = 0 \text{ which means } F_Y = W$$

Similarly

$$\sum F_x = 0 \Rightarrow F_{X1} + F_{X2} = 0 \text{ or } F_{X2} = -F_{X1}$$

To find, F_{X1} or F_{X2}, let us take moment about point A or B

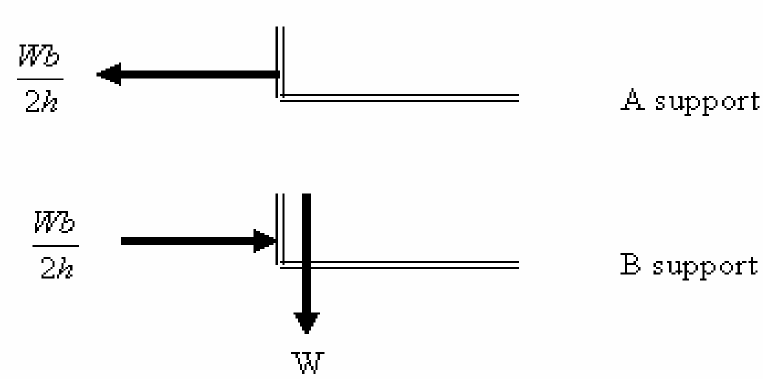
Let us make $\sum \tau_B = 0$. This gives (following the convention that counterclockwise torque is positive and clockwise torque is negative)

$$-F_{X1}h + W\frac{b}{2} = 0$$

$$\text{or } F_{X1} = \left(\frac{Wb}{2h}\right)$$

$$\text{and } F_{X2} = -\left(\frac{Wb}{2h}\right)$$

The negative sign for F_{X2} means that the force's direction is opposite to what it was taken to be in figure . We also wish to find the forces and couple on the support. By Newton 's IIIrd Law, forces on the support are opposite to those on the gate. Thus the forces on the two supports are:



Forces on the two supports

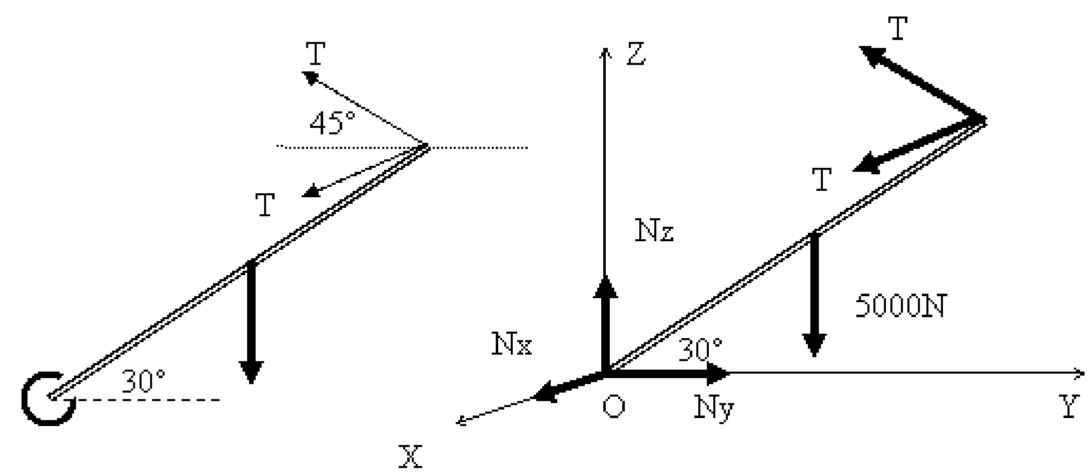
Figure

You see that support A is being pulled out whereas support B is being pushed in (we observe an effect of this at our houses all the time: the upper hinges holding a door tend to come out of the doorjamb). Now the force by the wall on support A will be $\left(\frac{Wb}{2h}\right)$ to the right to keep it fixed in its place. On the other hand the situation for the lower support is more involved. The lower support will be kept in its place by the wall providing it horizontal and vertical forces and a torque. The net horizontal force is $\left(\frac{Wb}{2h}\right)$ to the left and the net vertical force is W pointing up. The lower support also balances a torque. Taking the torques about the point where it enters the wall, its value comes out to be

$$\tau = Wa$$

Q.12 To balance a heavy weight of 5000 *N*, two persons dig a hole in the ground and put a pole of length *l* in it so that the hole acts as a socket. The pole makes an angle of 30° from the ground. The weight is tied at the mid point of the pole and the pole is pulled by two horizontal ropes tied at its ends as shown in figure . Find the tension in the two ropes and the reaction forces of the ground on the pole.

Ans. :-



A pole balancing a weight on it (left). Forces acting on it are shown on the right.

Figure

To solve this problem, let me first choose a co-ordinate system. I choose it so that the pole is over the y-axis in the *(y-z)* plane (see figure).

The ropes are in *(xy)* direction with tension T in each one of them so that tension in each is written as

$$\left(\frac{T}{\sqrt{2}}\hat{i} - \frac{T}{\sqrt{2}}\hat{j}\right) \quad \text{and} \quad \left(-\frac{T}{\sqrt{2}}\hat{i} - \frac{T}{\sqrt{2}}\hat{j}\right)$$

You may be wondering why I have taken the tension to be the same in the two ropes. Actually it arises from the torque balance equation; if the tensions were not equal, their component in the x-direction will give a nonzero torque.

Let the normal reaction of the ground be (Nx , Ny , Nz). Then the force balance equation gives

$$\sum F_x = 0 \Rightarrow N_x + \frac{T}{\sqrt{2}} - \frac{T}{\sqrt{2}} = 0 \Rightarrow N_x = 0$$

$$\sum F_y = 0 \Rightarrow N_y - \frac{2T}{\sqrt{2}} = 0 \Rightarrow N_y = T\sqrt{2}$$

$$\sum F_z = 0 \Rightarrow N_z - 5000 = 0 \Rightarrow N_z = 5000N$$

Taking torque about point O and equating it to zero, we get

$$\left(l\cos 30^o\,\hat{j} + l\sin 30^o\,\hat{k}\right)\times\left(-T\sqrt{2}\,\hat{j}\right) + \left(\frac{l}{2}\cos 30^o\,\hat{j} + \frac{l}{2}\sin 30^o\,\hat{k}\right)\times\left(-5000\hat{k}\right) = 0$$

$$Tl\sqrt{2}\,\frac{1}{2}\hat{i} - 2500\cdot l\cdot\frac{\sqrt{3}}{2}\hat{i} = 0 \quad \Rightarrow \quad T\sqrt{2} = 2500\sqrt{3}$$

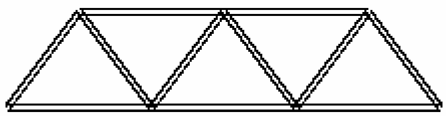
which gives

$$\boxed{T = 3062N. \qquad N_y = 4330N}$$

Chapter - 3

Q.1 What do you understand by trusses?

Ans. :- Having set up the basics for studying equilibrium of bodies, we are now ready to discuss the trusses that are used in making stable load-bearing structures. The examples of these are the sides of the bridges or tall TV towers or towers that carry electricity wires. Schematic diagram of a structure on the side of a bridge is drawn in figure.



Side of a bridge

Figure

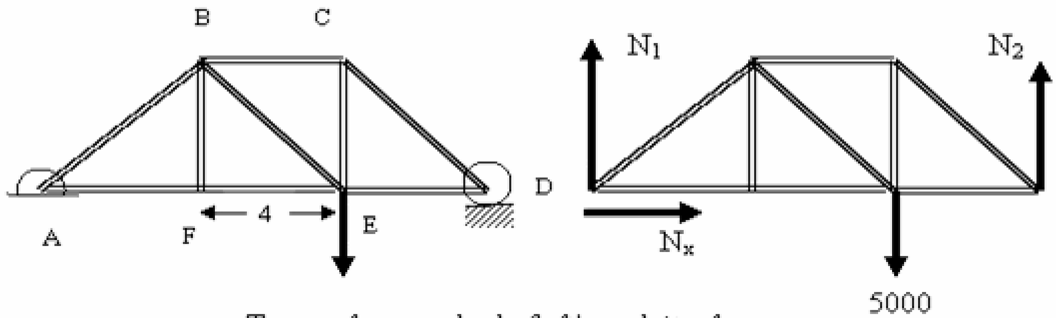
The structure shown in figure is essentially a two-dimensional structure. This is known as a plane truss. On the other hand, a microwave or mobile phone tower is a three-dimensional structure. Thus there are two categories of trusses - Plane trusses like on the sides of a bridge and space trusses like the TV towers.

Q.2Tell about method of Joints to analyze a truss?

Ans. - **Method of joints:** In method of joints, we look at the equilibrium of the pin at the joints. Since the forces are concurrent at the pin, there is no moment equation and only two equations for equilibrium viz. $\sum F_x = 0$ and $\sum F_y = 0$. Therefore we start our analysis at a point where one known load and at most two unknown forces are there. The weight of each member is divided into two halves and that is supported by each pin.

Q.3 Take truss ABCDEF as shown in figure 6 and load it at point E by 5000N. The length of small members of the truss is 4m and that of the diagonal members is $4\sqrt{2}$ m. Now find the forces in each member of this truss assuming them to be weightless.

Ans. -



Truss of example 1 (left) and its free body diagram (right)

Figure

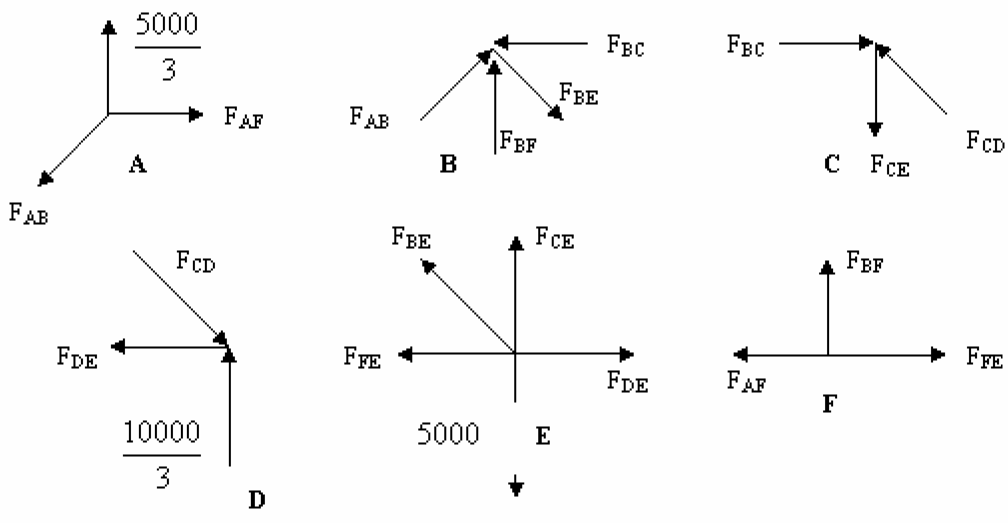
We take each point to be a pin joint and start balancing forces on each of the pins. Since pin E has an external load of 5000N one may want to start from there. However, E point has more than 2 unknown forces so we cannot start at E. We therefore first treat the truss as a whole and find reactions of ground at points A and D because then at points A and D their will remain only two unknown forces. The horizontal reaction *Nx* at point A is zero because there is no external horizontal force on the system. To find N_2 I take moment about A to get

$$N_2 = \frac{10000}{3} N$$

which through equation $\sum F_y = 0$ gives

$$N_1 = \frac{5000}{3} N$$

In method of joints, let us now start at pin A and balance the various forces. We already anticipate the direction and show their approximately at A (figure 7). All the angles that the diagonals make are 45° .



Forces at various joints of the truss in figure

Figure

The only equations we now have worry about are the force balance equations.

$$\sum F_y = 0 \quad gives \quad \frac{F_{AB}}{\sqrt{2}} = \frac{5000}{3} \quad or \quad F_{AB} = 2355N$$

$$now \sum F_x = 0 \quad gives \quad F_{AF} = \frac{F_{AB}}{\sqrt{2}} = \frac{5000}{3} N$$

Keep in mind that the force on the member AB and AF going to be opposite to the forces on the pin (Newton 's IIIrd law). Therefore force on member AB is compressive (pushes pin A away) whereas that on AF is tensile (pulls A towards itself).

Next I consider joint F where force AF is known and two forces BF and FE are unknown. For pint F

$$\sum F_x = 0 \quad gives \quad F_{FE} = F_{AF} = \frac{5000}{3} (tensile)$$

$$\sum F_y = 0 \quad gives \quad F_{BF} = 0 (No\, Force\, on\, BF)$$

Next I go to point B since now there are only two unknown forces there. At point B

$$\sum F_y = 0 \quad gives \quad F_{AB}\cos 45^o + F_{BE}\cos 45^o = 0$$
$$\text{or} \quad F_{BE} = -F_{AB} = -2355N$$

Negative sign shows that whereas we have shown F_{BE} to be compressive, it is actually tensile.

$$\sum F_x = 0 \quad \Rightarrow \quad F_{BC} - F_{AB}\sin 45^o - F_{BE}\sin 45^o = 0$$
$$F_{BC} = \frac{F_{AB}}{\sqrt{2}} + \frac{F_{BE}}{\sqrt{2}} = \frac{10,000}{3}N.(compressive)$$

Next I consider point C and balance the forces there. I have already anticipated the direction of the forces and shown F_{CE} to be tensile whereas F_{CD} to be compressive

$$\sum F_x = 0 \quad gives \quad F_{BC} = \frac{F_{CD}}{\sqrt{2}} \Rightarrow F_{CD} = F_{BC}\sqrt{2} = 4710N$$
$$\sum F_Y = 0 \quad gives \quad F_{CE} = \frac{F_{CD}}{\sqrt{2}} = \frac{F_{BC}\sqrt{2}}{\sqrt{2}} = F_{BC} = \frac{10,000}{3}N$$

Next I go to pin D where the normal reaction is $\frac{10,000}{3}$ N and balance forces there.

$$\sum F_Y = 0 \quad gives \quad \frac{F_{CD}}{\sqrt{2}} = \frac{10,000}{3}N$$
$$\sum F_x = 0 \quad gives \quad F_{DE} = \frac{F_{CD}}{\sqrt{2}} = \frac{10,000}{3}$$

Thus forces in various members of the truss have been determined. They are

$$F_{AB} = \frac{5000\sqrt{2}}{3}N(compressive), F_{AF} = \frac{5000}{3}N(Tensile), F_{BF} = 0$$
$$F_{FE} = \frac{5000}{3}N(Tensile), F_{BC} = \frac{10,000}{3}N(Compressive), F_{BE} = \frac{5000\sqrt{2}}{3}N(Tensile)$$
$$F_{CE} = \frac{10,000}{3}N(Tensile), F_{CD} = \frac{10,000\sqrt{2}}{3}N\ (compressive), F_{DE} = \frac{10,000}{3}N(Tensile)$$

Q.4 Explain method of Sections for analysis of trusses?

Ans. :- **Method of sections :** As the name suggests in method of sections we make sections through a truss and then calculate the force in the members of the truss though which the cut is made.

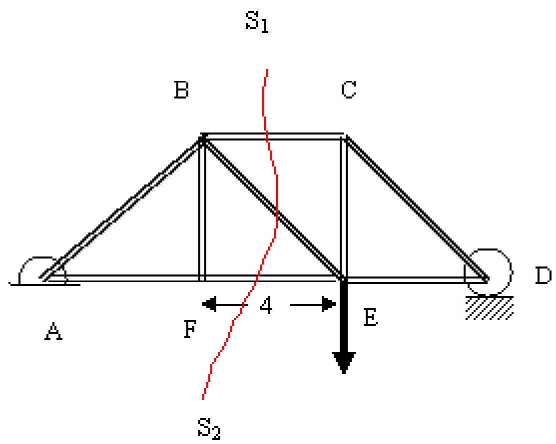
steps that are taken to solve for forces in members of a truss by method of sections:

1. Make a cut to divide the truss into section, passing the cut through members where the force is needed.
2. Make the cut through three member of a truss because with three equilibrium equations viz. $\sum F_x = 0, \sum F_y = 0$ and $\sum \tau = 0$ we can solve for a maximum of three forces.
3. Apply equilibrium conditions and solve for the desired forces.

Q.5 For example, if take the problem we just solved in the method of joints and make a section S_1, S_2 (see figure 9), we will be able to determine the forces in members BC, BE and FE by considering the equilibrium of the portion to the left or the right of the section?

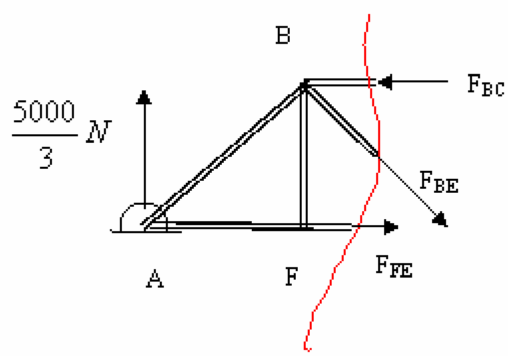
Ans. :- Let me now illustrate this. As in the method of joints, we start by first determining the reactions at the external support of the truss by considering it as a whole rigid body. In the present particular case, this gives $\frac{10000}{3}$ *N* at D and $\frac{5000}{3}$ *N* at A. Now let us consider the section of the truss on the left (see figure 10).

Since this entire section is in equilibrium, $\sum F_x = 0, \sum F_y = 0$ and $\sum \tau = 0$. Notice that we are now using all three equations for equilibrium since the forces in individual members are not concurrent. The direction of force in each member, one can pretty much guess by inspection. Thus the force in the section of members BE must be pointing down because there is no other member that can give a downward force to counterbalance $\frac{5000}{3}$ *N* reaction at A.



A cut made through a truss to apply the method of sections

Figure



Left section of the truss taken to apply method of sections

Figure

This clearly tells us that *F BE* is tensile. Similarly, to counter the torque about B generated by $\frac{5000}{3}$ *N* force at A, the force on FE should also be from F to E. Thus this force is also tensile. If we next consider the balance of torque about A, $\frac{5000}{3}$ N and F_{FE} do not give any torque about A. So to counter torque generated by F_{BE} , the force on BC must act towards B, thereby making the force compressive.

Let us now calculate individual forces. F_{FE} is easiest to calculate. For this we take the moment about B. This gives

$$4 \times \frac{5000}{3} = 4 \times F_{FE}$$

$$F_{FE} = \frac{5000}{3} N$$

Next we calculate F_{BE} . For this, we use the equation $\Sigma F_Y = 0$. It gives

$$\frac{F_{BE}}{\sqrt{2}} = \frac{5000}{3} N \quad \text{or} \quad F_{BE} = \frac{5000\sqrt{2}}{3} N$$

Finally to calculate F_{BC} , we can use either the equation $\Sigma \tau = 0$ about A or $\Sigma F_x = 0$

$$\sum F_x = 0 \quad \text{gives} \quad \frac{F_{BE}}{\sqrt{2}} + F_{FE} = F_{BC}$$

$$\Rightarrow F_{BC} = \frac{5000}{3} + \frac{5000}{3} = \frac{10,000}{3} N$$